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Two-Channel Quadrature Mirror Filter Bank Design using FIR Polyphase Component

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***Abstract*-This paper presents an efficient method for the design of two-channel quadrature mirror filter (QMF) bank by polyphase representation of low-pass prototype filter and power optimization method for minimization of an objective function. The objective function is formulated as linear combination of pass-band error and stop-band residual energy of the low-pass analysis filter of the filter bank. Our simulation results show that the use of polyphase components in the design of low-pass prototype filter reduces the computational complexity and the number of iterations. Two design examples are included to show the validity of the proposed method and its superiority over existing methods.**

***Keywords*—Filter banks, Polyphase component, Power optimization method, QMF bank, Eigenvector.**

I. Introduction

A two-channel QMF bank is extensively used in many signal processing fields. Initially, it was used for aliasing cancellation in sub-band coding of the speech and image signals [1-4]. Now a days the applications of QMF can be found in design of wavelet bases [5,6], speech and image compression [7,8], trans-multiplexers [9], antenna systems [10], digital audio industry [11], and biomedical signal processing [12] due to advancement in QMF banks. Because of such wide applications, many researchers giving a lot of attention in efficient design of such filter bank.

A typical two-channel QMF bank is shown in Fig. 1, the reconstructed signal $y(n)$ is not perfect replica of the input signal $x(n)$ due to three types of errors: aliasing distortion (ALD), phase distortion (PHD), and amplitude distortion (AMD). ALD can be cancelled completely by selecting the synthesis filters cleverly in terms of the analysis filters, whereas PHD eliminated using the linear phase FIR filters [1,17]. AMD is then minimized by optimizing the filter tap weights of the low-pass analysis filter using computer aided techniques. Several techniques for the design of QMF bank have been reported [8, 13-24].

The design problem for the QMF bank is to find the filters of the analysis/synthesis sections such that the reconstructed signal $y(n)$ approximates the original signal $x(n)$ in some optimal sense.

The z-transform of the output signal $y(n)$ of the two-channel QMF bank, is given by [13–15,19].

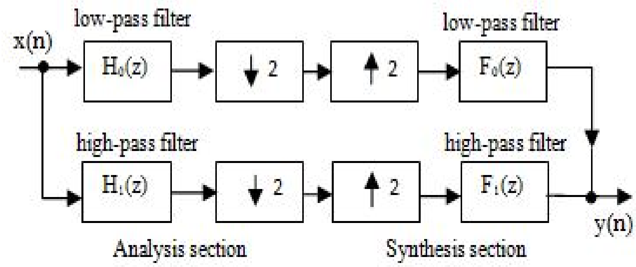

Fig. 1 Two-channel quadrature mirror filter bank

$$Y(z) = \tfrac{1}{2}\,[H_0(z)F_0(z) + H_1(z)F_1(z)]X(z) + \tfrac{1}{2}[\,H_0(-z)F_0(z) + H_1(-z)F_1(z)]\,X(-z) \quad (1)$$

aliasing can be completely removed by defining the synthesis filters as given below

$$F_0(z) = H_1(-z) \quad \text{and} \quad F_1(z) = -H_0(-z) \quad (2)$$

The relationship $H_1(z) = H_0(-z)$ between the mirror image filters and by using "(2)," the expression for the alias free reconstructed signal can be written as:

$$\begin{aligned} Y(z) &= \tfrac{1}{2}\,[H_0(z)\,H_1(-z) - H_1(z)H_0(-z)]X(z) \\ &= \tfrac{1}{2}[\,H_0^2(z) - H_0^2(-z)]\,X(z) \end{aligned} \quad (3)$$

$$\text{or} \qquad Y(z) = T(z)X(z) \quad (4)$$

If the analysis filter $H_0(z)$ is selected to be a linear phase FIR then phase response of the transfer function , $T(z)$, of the QMF bank also become linear and phase distortion of the QMF bank is eliminated. The corresponding frequency response can be written as

$$H_0(e^{j\omega}) = e^{-j\omega(N-1)/2}H_R(\omega) \quad (5)$$

where $H_R(\omega)$ is the amplitude function. Hence, by substituting "(5)," the overall transfer function of the QMF bank can be written as

$$T(e^{j\omega}) = \tfrac{1}{2}(e^{-j\omega(N-1)})[|H_0(e^{j\omega})|^2 - (-1)^{N-1}|H_0(e^{j(\pi-\omega)})|^2] \quad (6)$$

If the length of filter, N, is odd, above equation gives $T(e^{j\omega}) = 0$ at $\omega = \pi/2$, implying severe amplitude distortion. Therefore, N must be chosen to be even to avoid this distortion. By "(6)," the condition for perfect reconstruction is given as

$$|H_0(e^{j\omega})|^2 + |H_0(e^{j(\pi-\omega)})|^2 = c \quad (7)$$

Now by "(3)," the design problem of filter bank reduces to the determination of the filter tap coefficients of the linear

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phase FIR low-pass analysis filter $H_0(z)$ only, subject to the perfect reconstruction condition of "(7),".

This paper proposes an improved method for the design of a two-channel QMF bank in which polyphase components are used for implementation of filter bank. The polyphase component based design of QMF bank is efficient in the sense of speed constraints imposed on the processors. The objective function, which is a linear combination of the pass-band error and stop-band residual energy of the low-pass analysis filter $H_0(z)$ is minimized using power optimization method. The results are compared with the other existing methods to demonstrate the performance of the proposed method.

The organization of the paper is as follows. Section 2 gives the brief description of the implementation of QMF bank through polyphase component. Section 3 describes the design problem formulation. Section 4 describes the proposed optimization method and design algorithm. Section 5 examines the performance of the filter bank designed and comparison with other existing methods.

II. QMF Bank Using Polyphase Component

The QMF bank can be realized efficiently by using polyphase structure [17], which enables us to rearrange the computation of the filtering operation. For decimation filter, if direct form implementation is used then only the even numbered O/P samples are computed and this computation requires (N+1) multiplication per unit time (MPUs) and N addition per unit time (APUs). However during the computation of odd numbered O/P samples, the structure is merely resting. If we use polyphase implementation then the computation of O/P samples requires (N+1)/2 MPUs and N/2 APUs [1]. The multipliers and adders in each of the filter $E_0(z)$ and $E_1(z)$ now have two units of time for doing their works and they are continually operative and no resting time. Thus polyphase representation of decimation filter, as shown in Fig. 2, reduces the computational complexity of the multipliers and adders in filter bank.

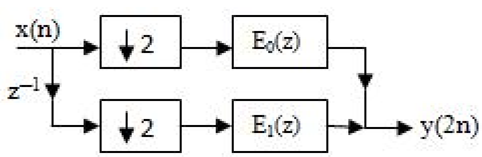

Fig. 2 Polyphase implementation of decimation filter

The prototype filter $H_0(z)$ can be written in its polyphase form as

$$H_0(z) = E_0(z^2) + z^{-1}E_1(z^2) \tag{8}$$

and

$$E_1(z) = z^{-\frac{N-1}{2}}E_0(z^{-1}) \tag{9}$$

by putting the value of $E_1(z)$ in to "(8)," we get

$$H_0(z) = E_0(z^2) + z^{-N}E_0(z^{-2}) \tag{10}$$

or

$$H_0(e^{j\omega}) = E_0(e^{2j\omega}) + e^{-jN\omega}E_0(e^{-2j\omega}) \tag{11}$$

where

$$E_0(e^{2j\omega}) = \sum_{n=0}^{M=(N-1)/2} e_0(n)e^{-2j\omega n} \tag{12}$$

and

$$E_0(e^{-2j\omega}) = \sum_{n=0}^{M=(N-1)/2} e_0(n)e^{2j\omega n} \tag{13}$$

so, the equation of prototype filter can be written as:

$$H_0(e^{j\omega}) = \sum_{n=0}^{M} e_0(n)e^{-2j\omega n} + e^{-jN\omega}\sum_{n=0}^{M} e_0(n)e^{2j\omega n} \tag{14}$$

The amplitude response of prototype filter can be given as:

$$|H(e^{j\omega})|^2 = |(\sum_{n=0}^{M} p_n\, e^{-2j\omega n} + e^{-jN\omega}\sum_{n=0}^{M} p_n\, e^{2j\omega n})|^2$$

or

$$|H(e^{j\omega})|^2 = |\sum_{n=0}^{M} p_n\left\{e^{-\frac{j\omega N}{2}}\left(\sum_{n=0}^{M} e^{-j\omega\left(2n-\frac{N}{2}\right)}\right) + \left(\sum_{n=0}^{M} e^{j\omega\left(2n-\frac{N}{2}\right)}\right)\right\}|^2 \tag{15}$$

$$\left|H(e^{j\omega})\right|^2 = |\sum_{n=0}^{M} p_n\{e^{-\frac{j\omega N}{2}}\cos(\omega\left(2n-\frac{N}{2}\right))\}|^2 = \mathbf{p^T c c^T p} \tag{16}$$

where vectors **p** and **c** are:

$$\mathbf{p} = [p_0\ p_1\ p_2\ \ldots\ldots p_M]^T \tag{17}$$

$$\mathbf{c} = [\cos\omega\left(-\frac{N}{2}\right)\ \cos\omega\left(2-\frac{N}{2}\right)\ldots\ldots\cos\omega\left(2M-\frac{N}{2}\right)]^T \tag{18}$$

The vectors **p** and **c** will be used for the designing of objective function in the next section.

III. Formulation of Design Problem

The objective function ϕ to be minimized for the design of QMF bank using polyphase components can be chosen as a linear combination of two functions:

$$\phi = \alpha E_s + (1-\alpha)E_p \tag{19}$$

The constant α is real and in the range $0 \le \alpha \le 1$, E_s and E_p are the measure of stop band residual energy and pass band error respectively. We propose to minimize the above objective function, by optimizing the coefficients of low pass filter $H_0(z)$. The calculation of stop band error and pass band error is given below.

A. Pass band Error (E_p) :

The amplitude response at zero frequency is given by

$$H_R(0) = \mathbf{p^T 1} \tag{20}$$

where **1** is the vector of all 1's. By taking this as a reference then the pass band deviation at any frequency can be given as:

$$\mathbf{p^T 1} - \mathbf{p^T c} = \mathbf{p^T(1-c)} \tag{21}$$

where vector **c** is defined in "(18)," so the pass band error can be written as:

$$E_p = \mathbf{p^T Q p} \tag{22}$$

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where

$$Q = \frac{1}{\pi}\int_0^{\omega_p}(\mathbf{1}-\mathbf{c})(\mathbf{1}-\mathbf{c})^T\, d\omega \tag{23}$$

B. Stop band Error (E_s):

The stop band error E_s is given as:

$$E_s = \frac{1}{\pi}\int_{\omega_s}^{\pi} |H(e^{j\omega})|^2\, d\omega = \mathbf{p}^T\, \mathbf{S}\, \mathbf{p} \tag{24}$$

where

$$\mathbf{S} = \frac{1}{\pi}\int_{\omega_s}^{\pi} \mathbf{c}\mathbf{c}^T\, d\omega \tag{25}$$

vector **c** is defined in "(18)," and by using the value of E_p and E_s the objective function can be written as :

$$\phi = \mathbf{p}^T \mathbf{R}\, \mathbf{p} \tag{26}$$

where

$$\mathbf{R} = \alpha \mathbf{S} + (1-\alpha)\mathbf{Q} \tag{27}$$

Here matrix **R** is a real, symmetric and positive definite matrix. The unit norm vector **p** which minimizes the objective function ϕ is the eigenvector corresponding to the minimum eigenvalue, λ_{min}, of matrix **R** which is calculated using 'Power method' described in next section.

IV. Power Optimization Method and Design Algorithm

The power method [1] is a simple iterative method which is designed to compute the dominant eigenvalue and corresponding eigenvector of the matrix. The other standard methods for finding the eigenvalues of a matrix **R** are impracticable when **R** is large. Even evaluating the determinant of an n × n matrix is a huge task when n is large. The principle of power method is that the vectors multiplied by matrix **R**, stretched in the direction of eigenvalue. In this method, we start with an initial vector $\mathbf{h_0}$ chosen arbitrarily and generate a sequence of approximations to get optimized filter coefficients. Here the iterative steps are performed a total of *n* times unless the vector $\mathbf{h_0}$ is orthogonal to every vector in the eigenspace. The matrix **R** contains complete set of eigenvectors with corresponding eigenvalues. This eigenvector corresponding to minimum eigenvalue contains the final optimized filter coefficients.

Design Algorithm:

Step 1: Guess an eigenvector $\mathbf{h_o}$ and normalize it as:

$$\mathbf{y_0} = \mathbf{h_0}/\|\mathbf{h_0}\|$$

Step2: Assume the length of the filter (*N*).

Step 3: Calculate inverse of matrix **R** and put it equal to $\mathbf{R_1}$.

Step 4: Multiply $\mathbf{y_0}$ by $\mathbf{R_1}$ to get a new vector. Normalize the result and call it as $\mathbf{y_1} = \mathbf{y_0} . \mathbf{R_1}$.

Step 5: Repeat the process *n* times so that

$$\mathbf{R_1 y_{n-1}} = \mathbf{R_1 y_n} = \boldsymbol{\lambda} \mathbf{y_n}$$

Hence generated vector $\mathbf{y_n}$ is the eigenvector of matrix '**R**' which minimizes the objective function and $1/\lambda$ is the minimum eigenvalue (λ_{min}) of matrix '**R**'.

V. Results and Discussion

A MATLAB program has been written to implement the design procedure described in the previous section. Use of polyphase components with aliasing free condition makes the system more efficient by reducing the computational time and number of iterations (NOI). The comparison of the proposed method for the design of QMF bank with other existing method is given in Table1. Two design examples are presented to illustrate the effectiveness of the proposed method.

The performance is evaluated in terms of pass band error (E_p), stop band error (E_s), NOI, computational time (CPU time), stop-band first lobe attenuation (A_l), stop-band edge attenuation (A_s) = " 20 $\log_{10}(H_0(\omega_s))$ as given in [19] and peak reconstruction error (PRE) in dB. The filter coefficients initially taken are $h_0(n)$ = [0, 0, 0, 0……0, 0.707].

Example 1: For filter length (N) = 24, $\omega_s = 0.6\pi$, $\omega_p = 0.4\pi$ and $\alpha = 0.52$.

The optimized 12 filter coefficients of low pass analysis filter:

$h(n)$ = [-0.0025 0.0173 -0.0001 -0.0361 0.0076 0.0628 -0.0287 -0.1075 0.0771 0.2006 -0.2556 -0.9330]

The corresponding magnitude plots of analysis filters $H_0(z)$ and $H_1(z)$ are depicted in Fig. 3a. The attenuation characteristics of low-pass filter $H_0(z)$ is plotted in Fig. 3b. Fig. 3c and 3d represent the magnitude of distortion function $T(z)$ and reconstruction error of QMF bank (in dB) respectively. The significant parameters obtained are $E_p = 2.1975\times10^{-5}$, $E_s = 2.924\times10^{-5}$, A_s = 26.84 dB, A_l = 36 dB, NOI=02, PRE = 0.2249 dB, and CPU-time = 0.00295sec (on core 2 duo processor 2.1 GHz, 1 GB RAM).

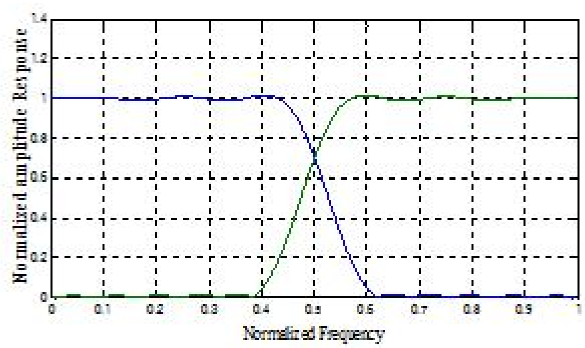

Fig. 3a Amplitude response of analysis filters for N =24

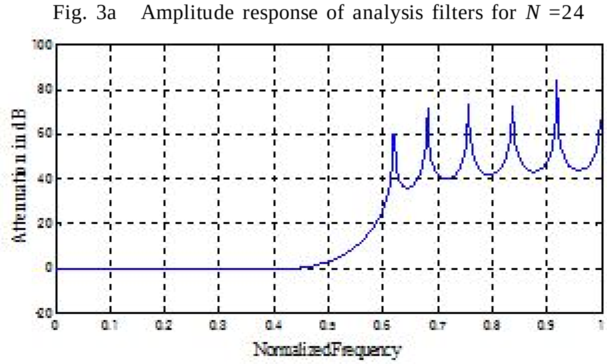

Fig. 3b Attenuation characteristics of low-pass analysis filter

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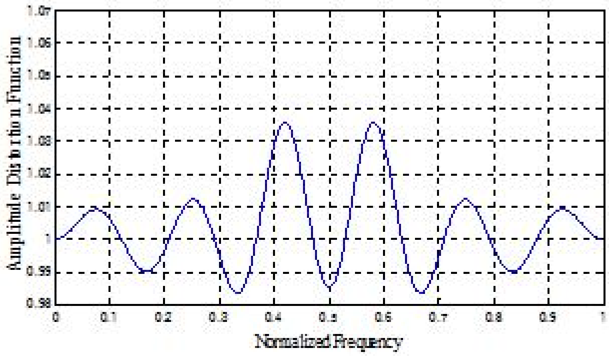

Fig. 3c Amplitude of the distortion function

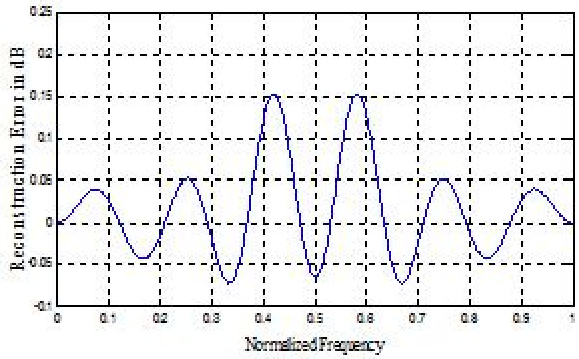

Fig. 3d Reconstruction error in dB

Example 2: For filter length (N) = 32, $\omega_s = 0.6\pi$, $\omega_p = 0.4\pi$ and $\alpha = 0.72$.

The optimized 16 filter coefficients of low pass analysis filter:

h(n) = [-0.0005 0.0062 0.0011 -0.0140 -0.0004 0.0255 -0.0041 -0.0430 0.0141 0.0682 -0.0360 -0.1096 0.0850 0.1998 -0.2620 -0.9291]

The corresponding magnitude plots of analysis filters $H_0(z)$ and $H_1(z)$ are shown in Fig. 4a. The attenuation characteristics of low pass filter $H_0(z)$ is plotted in Fig. 4b. Fig. 4c and 4d represent the magnitude of distortion function $T(z)$ and reconstruction error of QMF bank (in dB) respectively. The significant parameters obtained are $E_p = 4.227\times10^{-7}$, $E_s = 1.98\times10^{-6}$, $A_s = 37$ dB, $A_l = 45.86$ dB, NOI = 03, PRE = 0.1515 dB, and CPU-time = 0.00663sec.

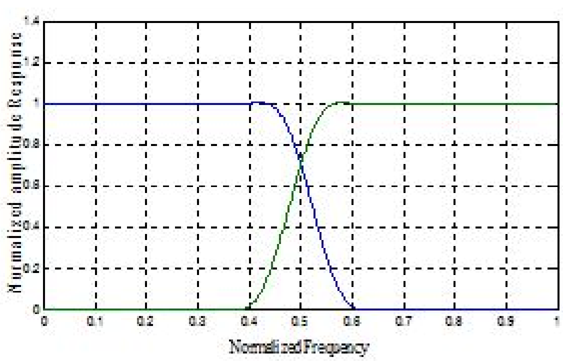

Fig. 4a Amplitude response of analysis filters for N=32

Table I summarizes that the proposed method gives improved performance than all other existing methods in terms of number of iteration (NOI) required and the computational time of the processor. Therefore the proposed method is computationally efficient.

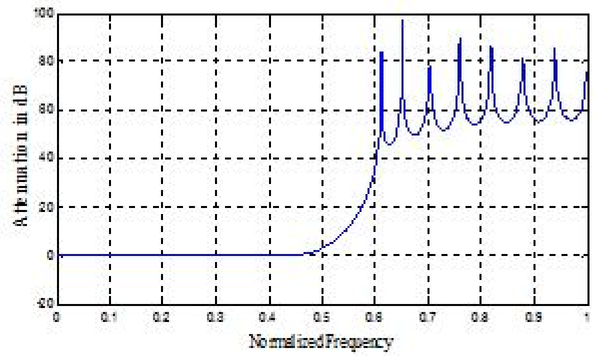

Fig. 4b Attenuation characteristics of low-pass analysis filter (N=32)

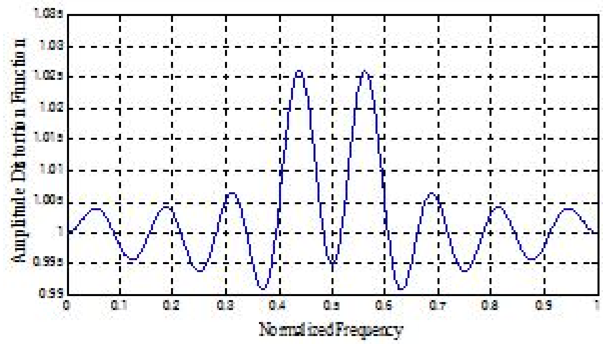

Fig.4c Amplitude of the distortion function (N=32)

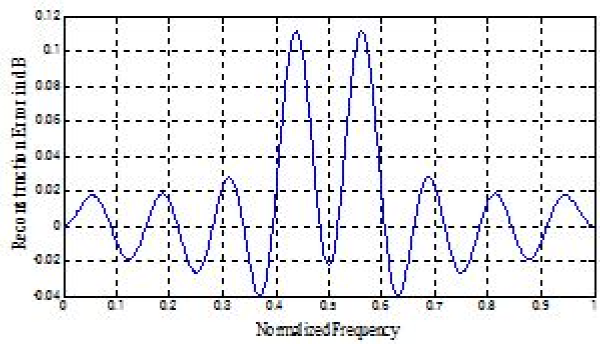

Fig. 4d Reconstruction error in dB (N=32)

TABLE I. PERFORMANCE COMPARISON OF PROPOSED METHOD WITH OTHER EXISTING METHODS FOR N = 32

Methods	E_p	E_s	A_s(dB)	A_l(dB)	NOI	CPU time (s)
Jain-Crochiere [14]	2.30 ×10[illegible]	1.50× 10^{-6}	33	44.25	30	0.245
Chen-Lee [13]	2.11 ×10[illegible]	1.55× 10^{-6}	34	35	26	0.210
A. Kumar-G. K. Singh [22]	7.42×10[illegible]	1.27× 10^{-6}	36.59	43.5	31	0.0152
O. P. Sahu [19]	1.45×10[illegible]	2.76× 10^{-6}	33.93	44.25	120	0.425
J. Upender et al. [21]	2.351×10[illegible]	5.79× 10^{-3}	36.87	44.75	68	0.0893
Proposed Method	4.227×10[illegible]	1.98× 10^{-6}	37	45.86	03	0.0066

CONCLUSION

In this paper, a new technique for designing of quadrature mirror filter (QMF) bank using polyphase component has

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been presented. The design problem minimizes a weighted sum of pass band error and stop band residual energy of the low-pass analysis filter. The comparison of the simulation results indicates that the proposed method requires less computational time (few msec. on core 2 duo processor 2.1 GHz, 1 GB RAM). The Polyphase decomposition results dramatic computational efficiency and reduces the number of iterations. This approach can be extended to the case of two-dimensional filters and for filters with larger taps.

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